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# *Optimal Design of Round Bottomed Triangle Channels*

**Ayman T. Hameed, Assistant Lecturer**

**Department of Civil Engineering, College of Engineering, University of Mosul**

## Abstract

In optimal design concept, the geometric dimensions of a channel cross-section are determined in a manner to minimize the total construction costs. The Direct search optimization method by using MATALAB is used to solve the resulting channel optimization models for a specified flow rate, roughness coefficient and longitudinal slope. The developed optimization models are applied to design the round bottomed triangle channel and trapezoidal channels to convey a given design flow considering various design scenarios However, it also can be extended to other shapes of channels. This method optimizes the total construction cost by minimizing the cross-sectional area and wetted perimeter per unit length of the channel. In the present study, it is shown that for all values of side slope, the total construction cost in the round bottomed triangle cross-section are less than those of trapezoidal cross-section for the same values of discharge. This indicates that less excavation and a lining are involved and therefore implies that the round bottomed triangle cross-section is more economical than trapezoidal cross-section.

**Keywords: Open channels; Optimization; Effective cost; Channel design; Direct search**.

# التصميم الأمثل للقنوات المثلثة ذات القعر الدائري

## الخلاصة

في مفهوم التصميم الأمثل، الأبعاد الهندسية تحسب بأسلوب إيجاد الحد الأدنى لكلف الإنشاء الكلية. طريقة البحث المباشر باستخدام برنامج ماتلاب استخدمت لحل موديلات القيم المثلى الناتجة للقنوات ذات القيم المعروفة للتصريف ومعامل الخشونة والميل الطولي للقناة. موديلات القيم المثلى التي ظهرت تم تطبيقها لتصميم مقطع القناة المثلثة ذات القعر الدائري وذات المقطع شبه المنحرف لنقل كمية التصريف المعطى باعتماد سيناريوهات تصميم مختلفة، من ناحية ثانية يمكن تطبيق هذا الأسلوب على أنواع أخرى من القنوات. هذه الطريقة توجد الحد الأدنى لكلف الإنشاء الكلية عن طريق تقليل مساحة المقطع العرضي والمحيط المبتل لكل وحدة طول من القناة. في هذه الدراسة تم ملاحظة أن لكل قيم الميل الجانبي كلفة الإنشاء الكلية في مقطع القناة المثلثة ذات القعر الدائري اقل من مقطع قناة شبه المنحرف لنفس القيم من التصريف،ذلك يتضمن على كميات حفر وتبطين اقل وعليه ضمنياً إن مقطع القناة المثلثة ذات القعر الدائري أكثر اقتصادية من مقطع قناة شبه المنحرف.

**الكلمات الدالة: القنوات المفتوحة، الامثلية، الكلفة الفعالة، تصميم القناة، البحث المباشر**

## Notations

$A_f$ =total cross-sectional area of the channel
A =wetted cross-sectional area of the channel
$P_f$ =total perimeter of the channel
P = wetted perimeter of the channel
$T_f$ =total top width of the channel
T =top width of the channel
f = freeboard of the channel
y =design depth of the channel
z =side slope of the channel
r =round bottomed of the triangle channel
b =bottom width of the trapezoidal channel
Q =design discharge of the channel
n =Manning's coefficient of roughness
S =longitudinal slope of the channel
V =velocity of flow of the channel
$C_1$ =excavation cost per unit volume of the channel
$C_2$=lining cost per unit area of the channel

## Introduction

The main function in the design of open channels for a given value of discharge, roughness coefficient and longitudinal slope is to find the optimal channel dimensions to convey the required discharge with minimum construction cost. Which can be determining using optimization techniques? Artificial channels are usually designed with section of regular geometric shapes. The trapezoid is the commonest shape for channels, the round bottomed triangle is an approximation of the parabola, it is a form usually created excavation with shovels (Chow, V.T. 1959) [1]. (Loganathan, G.V.1991)[2] Presented optimality condition for a parabolic-canal design accounting for freeboard and limitations on velocity and canal dimension. (Monadjemi, P.1994)[3] used undetermined multipliers method of Lagrange to find the best hydraulic cross sections for rectangular, triangular, trapezoidal and round bottom triangular channels. (Froehlich, D.C.1994)[4] Solved the problem of the finding the dimensions of the best hydraulic section of trapezoidal shape by using of an augmented Lagrange function for the case of no width or depth constructions. (Das, A.2000)[5] obtained optimal channel cross section with composite roughness for trapezoidal channel section by using Lagrange's multipliers to minimize the total construction cost of the channel for deferent lining materials for side slopes and bed of the channel. The equivalent uniform roughness coefficient was calculated using the equation proposed by Horton (1933). ( Babaeyan-Koopaei, K. et al. 2000)[6] Determined by using the undetermined multipliers method of Lagrange the hydraulic cross section of parabolic bottomed triangle, round bottomed triangle, parabolic and trapezoidal canals. (Jain, A. et al. 2004)[7] Proposed nonlinear optimization program, which consists of an objective function of minimizing total construction cost and dimensions of channel with composite roughness coefficient. The nonlinear optimization program was solved by using genetic algorithm. (Bhathachariya, R.K.2006)[8] Presented a nonlinear optimization model for designing an optimal channel section incorporating the critical flow condition of the channel. The objective of the optimization model to determine the cost effective channel section. The developed optimization model solved by sequential quadratic programming using MATLAB. ( Bhattachariya, R.K.and Satish, M.G.2007)[9] developed a new methodology to design a stable and optimal channel section using hybrid optimization techniques. A genetic algorithm based optimization model was developed to determine the factor of safety of channel slope for a given soil parameters. (Das, A.2007)[10] proposed a new cross-sectional shape that has a horizontal bed and two parabolic sides to minimizing the total construction cost. The developed optimization models to design the proposed and trapezoidal channels using Lagrange multiplier technique to solve the resulting channel optimization models. ( Bhattachariya, R.K.and Satish, M.G.2008)[11] presented a formulation for optimal design of an open channel section using freeboard as a design variable. The formulated multi-objective optimization model has been solved using the classical optimization method and nondominated sorting genetic algorithm. (Das, A.2008)[12] presented a mathematical model for chance constrained optimal design of trapezoidal channels. The developed model to maximize the probability of channel capacity and minimize the probable cost of the channels having composite and uniform roughness. (Hussein, A.S.A.2008)[13] estimated the wetted perimeter with freeboard using the isoperimetric theorem which results in a simple and accurate expression for the wetted perimeter that dose not lead to discontinuity in the optimal solution.

In the present study the problem of the nonlinear optimization model is to minimize the total construction costs of the channel. The total construction costs include excavation and lining costs per unit length of the channel. The nonlinear optimization model is solved by the direct search method in MATLAB.

## Problem Formulation

The round-bottomed triangle cross-section, as shown in Fig. (1) with side slope of z: 1 (H: V), the manning's roughness coefficient, flow depth, freeboard of the channel and radius of the round bottomed are (n, y, f, r).

Let A, P and T are the cross-sectional area, wetted perimeter and flow top width respectively. These hydraulic parameters can be written as

$$A = \frac{\left[2\left(z(y-r)+r\sqrt{1+z^2}\right)\right]^2}{4z} - \frac{r^2}{z}\left(1-z\cot^{-1}z\right) \text{..................................(1)}$$

$$P = \frac{2\left(z(y-r)+r\sqrt{1+z^2}\right)}{z} * \sqrt{1+z^2} - \frac{2r}{z}\left(1-z\cot^{-1}z\right) \text{..................................(2)}$$

$$T = 2\left(z(y-r)+r\sqrt{1+z^2}\right) \text{..................(3)}$$

Similarly, $A_f$ , $P_f$, and $T_f$ are the cross sectional area, wetted perimeter and flow top width with freeboard. These parameters can be written as:

$$A_f = \frac{\left[2\left(z((y+f)-r)+r\sqrt{1+z^2}\right)\right]^2}{4z} - \frac{r^2}{z}\left(1-z\cot^{-1}z\right) \text{..................................(4)}$$

$$P_f = \frac{2\left(z\left((y+f)-r\right)+r\sqrt{1+z^2}\right)}{z} * \sqrt{1+z^2} - \frac{2r}{z}\left(1-z\cot^{-1}z\right) \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$T_f = 2\left(z\left((y+f)-r\right)+r\sqrt{1+z^2}\right) \ldots\ldots\ldots(6)$$

The Manning's equation for uniform flow in an open channel is written as (Chow 1959)

$$\frac{Qn}{\sqrt{S}} - \frac{A^{5/2}}{P^{2/3}} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Where Q=design discharge of the channel; S=longitudinal slope of the channel.

## Direct Search Method

Direct search is a method for solving optimization problems that does not require any information about the gradient of the objective function. As opposed to more traditional optimization methods that use information about the gradient or higher derivatives to search for an optimal point. A direct search algorithm searches a set of points around the current point, looking for one where the value of the objective function is lower than the value at the current point. It can be used this method to solve problems for which the objective function is not differentiable, stochastic, or even continues [14].

## Proposed Nonlinear Optimization Model

The proposed nonlinear optimization model minimizes the total construction costs per unit length of the open channel. The construction costs include excavation and lining costs per unit length of the channel. The constraints are: (1) Manning's equation as an equality constraint; (2) an inequality constraint of side slope of the channel more than or equal to the minimum side slope and less than or equal to the maximum side slope of the channel; (3) the maximum velocity less than or equal to the average velocity. The nonlinear optimization model may be written as:

Minimize

$$J(y,r,z) = C_1\left\{\begin{array}{l}\dfrac{\left[2\left(z\left((y+f)-r\right)+r\sqrt{1+z^2}\right)\right]^2}{4z} - \\ \dfrac{r^2}{z}\left(1-z\cot^{-1}z\right)\end{array}\right\} + C_2\left\{\begin{array}{l}\dfrac{2\left(z\left((y+f)-r\right)+r\sqrt{1+z^2}\right)}{z} * \sqrt{1+z^2} \\ -\dfrac{2r}{z}\left(1-z\cot^{-1}z\right)\end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Subject to:

$$\frac{Qn}{\sqrt{S}} - \frac{\left\{\begin{array}{l}\dfrac{\left[2\left(z(y-r)+r\sqrt{1+z^2}\right)\right]^2}{4z} - \\ \dfrac{r^2}{z}\left(1-z\cot^{-1}z\right)\end{array}\right\}^{5/3}}{\left\{\begin{array}{l}\dfrac{2\left(z(y-r)+r\sqrt{1+z^2}\right)}{z} * \sqrt{1+z^2} - 3 \\ \dfrac{2r}{z}\left(1-z\cot^{-1}z\right)\end{array}\right\}^{2/3}} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

$$z_{\min.} \le z \le z_{\max.} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

$$v - v_{\max.} \le 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Where J= objective function equal to the total construction cost per unit length of the channel (in thousand Iraqi dinar units/m); $C_1$= excavation cost per unit volume of the channel (in thousand Iraqi dinar units/m$^3$); $C_2$ = lining cost per unit area of the channel (in thousand Iraqi dinar units/m$^2$); $z_{min.}$ equal to (0.514, 0.577, 1.000, and 1.500) to four scenarios and $z_{max.}$ equal to (1.750).

## Comparison With Common Cross-Section

Comparison between the cross-section parameters of a round bottomed triangle with trapezoidal has been carried out. The results of the comparison for T, P, A, V and y for different values of z and Q are given in tables (1) to (4). The comparison includes the total construction cost in addition to the cross-sectional parameters. The comparison shows that for all values of z and Q the cross-sectional area, wetted perimeter and top width of the round bottomed triangle cross-section are less than trapezoidal cross-section.

Below shown the objective function and constraints of the optimization model of trapezoidal cross-section.

Objective function (minimize) of trapezoidal cross-section

$$M(b,y,z)=C_1(b(y+f)+z(y+f)^2+C_2\left(b+2(y+f)*\sqrt{1+z^2}\right)\text{................}(12)$$

Subject to:

$$\frac{Qn}{\sqrt{S}}-\frac{\left(by+zy^2\right)^{5/3}}{\left(b+2y\sqrt{1+z^2}\right)^{2/3}}=0\text{.............}(13)$$

$$z_{\min.}\le z\le z_{\max.}\text{................................}(14)$$

$$v-v_{\max.}\le 0\text{......................................}(15)$$

## Problem Solution

The nonlinear optimization models formulated above are solved by the direct search method using MATLAB. Value of Manning's roughness coefficient is 0.015; a longitudinal channel slope of 0.001; a freeboard of 0.5 m; and cost elements $C_1$ and $C_2$ are 10 and 30 (thousand Iraqi dinar unit per meter) respectively, have been used in this study. In order to validate the models, the results obtained using those models are compared with the results obtained by ( Babaeyan-Koopaei, K. et al. 2000) [6] as shown in table (5).

## Conclusions and Recommendations

This paper presents a nonlinear optimization model for optimal design of round bottomed triangle cross-section and the most efficient hydraulic section is determined by using direct search method in MATLAP. The cross-sectional parameters of a round bottomed triangle cross-section are compared with a trapezoidal cannel section. The comparison shows that for all values of side slope and discharge, the area, wetted perimeter and top width in a round bottomed triangle cross-section are less than of trapezoidal. The smaller the cross-sectional area and the wetted perimeter, the lower are excavation and lining costs. There for the round bottomed triangle has been demonstrated to be more economical than trapezoidal channel. However, the solution results for the round bottomed triangle and trapezoidal section are compared with solution results obtained by ( Babaeyan-Koopaei, K. et al. 2000) [6]. And then it can be recommended extending the proposed method to other shapes of open channels such as parabolic, parabolic-bottomed triangle, circular and rectangular channels.

## Results and Discussion

The optimization results of the proposed model of round bottomed triangle channel was compared with the common cross-section of trapezoidal channel and the values of side slope $\begin{bmatrix}(0.514 \le z \le 1.75),(0.577 \le z \le 1.75),\\ (1.0 \le z \le 1.75),(1.5 \le z \le 1.75)\end{bmatrix}$, and different values of discharge, the results obtained shows the values of the total construction costs and cross-sectional parameters of round bottomed triangle channel are less than of trapezoidal channel, and from the figures (3), (4) it can be noted the values of total construction cost are increase with an increase in the values of side slope. Table (5) illustrate a comparison for solution results in this study with the results obtained by Babaeyan-Koopaei (2000), a comparative shows a good agreement of the obtained results with the other published results, for the same values of discharge, coefficient of roughness, longitudinal slope and side slope.

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tabulated in tables (1-4) for

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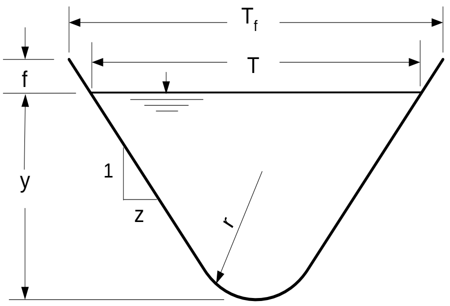

**Fig.(1) Round bottomed triangle channel cross-section**

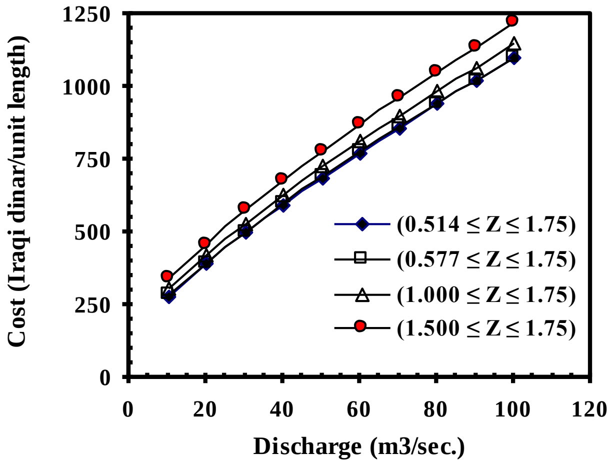

Fig. (3) Relation between cost and discharge forround bottomed triangle channel

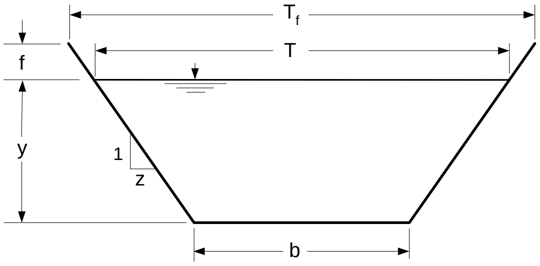

**Fig.(2) Trapezoidal channel cross-section**

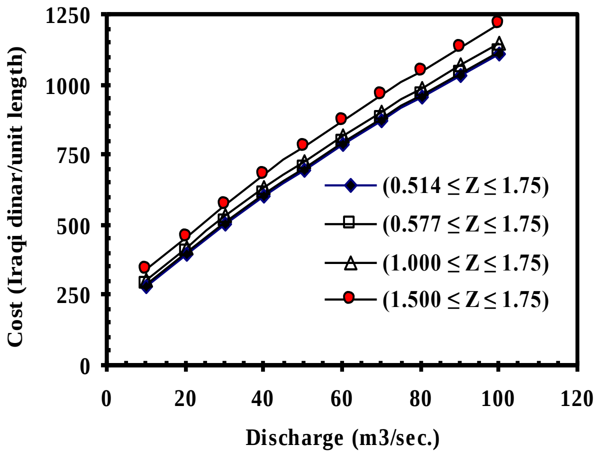

Fig. (4) Relation between cost and discharge for trapezoidal channel

## Table (1) Comparison of cross section parameters and cost between trapezoidal and round bottomed triangular channel - (0.514 $\subseteq$ 1.750)

| **Parameter** | **Q = 10 $m^3$/sec.** | | **Q = 20 $m^3$/sec.** | | **Q = 30 $m^3$/sec.** | | **Q = 40 $m^3$/sec.** | | **Q = 50 $m^3$/sec.** | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **S= 0.001** | **n= 0.015** | **S= 0.001** | **n= 0.015** | **S= 0.001** | **n= 0.015** | **S= 0.001** | **n= 0.015** | **S= 0.001** | **n= 0.015** |
| | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** |
| **z** | 0.516 | 0.514 | 0.514 | 0.514 | 0.514 | 0.514 | 0.514 | 0.514 | 0.514 | 0.514 |
| **y** | 1.785 | 1.843 | 2.470 | 2.538 | 3.020 | 3.205 | 3.500 | 3.648 | 3.900 | 4.000 |
| **b or r** | 2.001 | 1.651 | 2.586 | 2.200 | 3.178 | 2.500 | 3.643 | 2.991 | 4.100 | 3.505 |
| **P** | 6.018 | 5.747 | 8.140 | 7.842 | 9.969 | 9.634 | 11.513 | 11.107 | 12.870 | 12.397 |
| **A** | 5.215 | 5.120 | 9.523 | 9.519 | 14.285 | 14.277 | 19.047 | 19.041 | 23.807 | 23.801 |
| **T** | 3.843 | 3.910 | 5.125 | 5.294 | 6.282 | 6.346 | 7.241 | 7.401 | 8.1009 | 8.390 |
| **v** | 1.917 | 1.953 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 |
| **f** | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |
| **Cost** | 287.079 | 278.311 | 400.098 | 392.057 | 508.363 | 498.717 | 607.109 | 595.788 | 699.772 | 687.070 |

| **Parameter** | **Q = 60 $m^3$/sec.** | | **Q = 70 $m^3$/sec.** | | **Q = 80 $m^3$/sec.** | | **Q = 90 $m^3$/sec.** | | **Q = 100 $m^3$/sec.** | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **S= 0.001** | **n= 0.015** | **S= 0.001** | **n= 0.015** | **S= 0.001** | **n= 0.015** | **S= 0.001** | **n= 0.015** | **S= 0.001** | **n= 0.015** |
| | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** |
| **z** | 0.514 | 0.514 | 0.514 | 0.514 | 0.514 | 0.514 | 0.514 | 0.514 | 0.514 | 0.514 |
| **y** | 4.250 | 4.475 | 4.650 | 4.875 | 5.000 | 5.278 | 5.250 | 5.500 | 5.550 | 5.733 |
| **b or r** | 4.538 | 3.650 | 4.778 | 3.861 | 5.049 | 4.000 | 5.464 | 4.432 | 5.727 | 4.800 |
| **P** | 14.095 | 13.606 | 15.234 | 14.711 | 16.292 | 15.752 | 17.269 | 16.672 | 18.207 | 17.554 |
| **A** | 28.570 | 28.562 | 33.331 | 33.326 | 38.095 | 38.088 | 42.853 | 42.852 | 47.617 | 47.612 |
| **T** | 8.907 | 9.055 | 9.558 | 9.724 | 10.189 | 10.308 | 10.861 | 11.064 | 11.432 | 11.753 |
| **v** | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 |
| **f** | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |
| **Cost** | 788.124 | 774.264 | 873.190 | 858.354 | 955.694 | 940.131 | 1036.000 | 1019.000 | 1114.600 | 1096.600 |

## Table (2) Comparison of cross section parameters and cost between trapezoidal and round bottomed triangular channel - (0.577 ⊆ Z ⊆ 1.750)

| Parameter | Q = 10 $m^3$/sec. | | Q = 20 $m^3$/sec. | | Q = 30 $m^3$/sec. | | Q = 40 $m^3$/sec. | | Q = 50 $m^3$/sec. | |
|---|---|---|---|---|---|---|---|---|---|---|
| | S= 0.001 | n= 0.015 | S= 0.001 | n= 0.015 | S= 0.001 | n= 0.015 | S= 0.001 | n= 0.015 | S= 0.001 | n= 0.015 |
| | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle |
| **z** | 0.577 | 0.577 | 0.577 | 0.577 | 0.577 | 0.500 | 0.622 | 0.577 | 0.500 | 0.577 |
| **y** | 1.800 | 1.998 | 2.000 | 2.633 | 2.885 | 3.152 | 3.049 | 3.625 | 3.250 | 4.000 |
| **b or r** | 1.859 | 1.319 | 3.607 | 1.950 | 3.287 | 2.540 | 4.349 | 2.964 | 5.450 | 3.429 |
| **P** | 6.015 | 5.585 | 8.225 | 7.910 | 9.948 | 9.664 | 11.530 | 11.154 | 12.954 | 12.457 |
| **A** | 5.215 | 5.159 | 9.522 | 9.517 | 14.285 | 14.284 | 19.042 | 19.041 | 23.807 | 23.804 |
| **T** | 3.936 | 3.829 | 5.915 | 5.290 | 6.616 | 6.571 | 8.141 | 7.606 | 9.200 | 8.576 |
| **v** | 1.917 | 1.938 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 |
| **f** | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |
| **Cost** | 288.441 | 282.434 | 407.676 | 395.177 | 510.477 | 501.751 | 614.037 | 599.273 | 708.835 | 690.826 |

| Parameter | Q = 60 $m^3$/sec. | | Q = 70 $m^3$/sec. | | Q = 80 $m^3$/sec. | | Q = 90 $m^3$/sec. | | Q = 100 $m^3$/sec. | |
|---|---|---|---|---|---|---|---|---|---|---|
| | S= 0.001 | n= 0.015 | S= 0.001 | n= 0.015 | S= 0.001 | n= 0.015 | S= 0.001 | n= 0.015 | S= 0.001 | n= 0.015 |
| | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle |
| **z** | 0.577 | 0.577 | 0.577 | 0.577 | 0.577 | 0.577 | 0.577 | 0.577 | 0.577 | 0.577 |
| **y** | 3.529 | 4.500 | 3.975 | 4.758 | 4.285 | 5.000 | 4.500 | 5.299 | 5.000 | 5.654 |
| **b or r** | 6.057 | 3.503 | 6.092 | 4.000 | 6.417 | 4.471 | 6.927 | 4.750 | 6.638 | 4.855 |
| **P** | 14.205 | 13.646 | 15.270 | 14.744 | 16.311 | 15.746 | 17.317 | 16.698 | 18.183 | 17.617 |
| **A** | 28.561 | 28.560 | 33.332 | 33.315 | 38.091 | 38.093 | 42.855 | 42.841 | 47.615 | 47.610 |
| **T** | 10.129 | 9.390 | 10.679 | 10.110 | 11.361 | 10.934 | 12.120 | 11.601 | 12.408 | 12.132 |
| **v** | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 |
| **f** | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |
| **Cost** | 798.692 | 778.479 | 880.927 | 862.414 | 963.211 | 944.122 | 1044.800 | 1023.700 | 1119.800 | 1101.500 |

**Table (3) Comparison of cross section parameters and cost between trapezoidal and round bottomed triangular channel - (1.000 ⊆ Z ⊆ 1.750)**

| Parameter | Q = 10 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 20 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 30 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 40 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 50 $m^3$/sec.<br>S= 0.001 n= 0.015 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle |
| **Z** | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| **Y** | 1.788 | 1.750 | 2.350 | 2.382 | 2.950 | 2.917 | 3.350 | 3.370 | 3.750 | 3.765 |
| **b or r** | 1.169 | 1.588 | 1.702 | 2.040 | 1.892 | 2.500 | 2.335 | 2.880 | 2.599 | 3.230 |
| **P** | 6.226 | 6.127 | 8.348 | 8.250 | 10.235 | 10.106 | 11.810 | 11.668 | 13.205 | 13.045 |
| **A** | 5.287 | 5.254 | 9.522 | 9.517 | 14.283 | 14.280 | 19.044 | 19.036 | 23.808 | 23.796 |
| **T** | 4.745 | 4.815 | 6.402 | 6.453 | 7.792 | 7.905 | 9.035 | 9.125 | 10.099 | 10.205 |
| **V** | 1.891 | 1.902 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 |
| **F** | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |
| **Cost** | 308.420 | 305.442 | 422.662 | 420.036 | 533.840 | 530.555 | 634.914 | 631.201 | 729.691 | 725.538 |

| Parameter | Q = 60 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 70 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 80 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 90 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 100 $m^3$/sec.<br>S= 0.001 n= 0.015 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle | Trapezoidal | Round-Bottomed Triangle |
| **Z** | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| **Y** | 4.100 | 4.127 | 4.500 | 4.459 | 4.722 | 4.759 | 4.950 | 5.075 | 5.250 | 5.344 |
| **b or r** | 2.868 | 3.530 | 2.907 | 3.810 | 3.345 | 4.100 | 3.708 | 4.250 | 3.820 | 4.500 |
| **P** | 14.464 | 14.292 | 15.634 | 15.439 | 16.700 | 16.503 | 17.708 | 17.508 | 18.669 | 18.454 |
| **A** | 28.568 | 28.562 | 33.332 | 33.330 | 38.092 | 38.085 | 42.857 | 42.840 | 47.617 | 47.602 |
| **T** | 11.068 | 11.178 | 11.907 | 12.074 | 12.789 | 12.914 | 13.608 | 13.670 | 14.320 | 14.415 |
| **V** | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 |
| **F** | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Cost** | 819.939 | 815.393 | 906.856 | 901.852 | 990.870 | 985.623 | 1072.800 | 1067.200 | 1152.800 | 1146.900 |

### Table (4) Comparision of cross section parameter and caste between trapezoidal and round bottomed triangular channel $(1.500 \subseteq Z \subseteq 1.750)$

| Parameter | Q = 10 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 20 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 30 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 40 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 50 $m^3$/sec.<br>S= 0.001 n= 0.015 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** |
| **Z** | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 |
| **Y** | 1.6400 | 1.625 | 2.170 | 2.176 | 2.650 | 2.670 | 3.050 | 3.079 | 3.430 | 3.444 |
| **b or r** | 0.880 | 1.574 | 1.133 | 1.880 | 1.415 | 2.275 | 1.670 | 2.650 | 1.796 | 2.955 |
| **P** | 6.793 | 6.756 | 8.957 | 8.917 | 10.969 | 10.923 | 12.666 | 12.612 | 14.163 | 14.102 |
| **A** | 5.477 | 5.466 | 9.521 | 9.516 | 14.283 | 14.278 | 19.047 | 19.035 | 23.807 | 23.799 |
| **T** | 5.800 | 5.828 | 7.643 | 7.666 | 9.365 | 9.387 | 10.820 | 10.841 | 12.086 | 12.121 |
| **V** | 1.825 | 1.829 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 |
| **F** | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |
| **Cost** | 345.429 | 344.355 | 460.027 | 459.046 | 576.636 | 575.431 | 682.421 | 681.022 | 781.269 | 779.716 |

| Parameter | Q = 60 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 70 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 80 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 90 $m^3$/sec.<br>S= 0.001 n= 0.015 | | Q = 100 $m^3$/sec.<br>S= 0.001 n= 0.015 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** |
| **Z** | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 |
| **Y** | 3.800 | 3.771 | 4.075 | 4.099 | 4.350 | 4.426 | 4.650 | 4.673 | 4.800 | 4.917 |
| **b or r** | 1.818 | 3.250 | 2.067 | 3.350 | 2.232 | 3.320 | 2.241 | 3.650 | 2.720 | 3.900 |
| **P** | 15.519 | 15.450 | 16.759 | 16.689 | 17.916 | 17.851 | 19.006 | 18.930 | 20.26 | 19.950 |
| **A** | 28.568 | 28.566 | 33.331 | 33.318 | 38.092 | 38.086 | 42.854 | 42.848 | 47.616 | 47.595 |
| **T** | 13.218 | 13.281 | 14.292 | 14.325 | 15.282 | 15.288 | 16.191 | 16.229 | 17.120 | 17.110 |
| **V** | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 | 2.100 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **F** | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |
| **Cost** | 875.238 | 873.511 | 965.431 | 963.606 | 1052.700 | 1050.800 | 1137.600 | 1135.500 | 1220.400 | 1218.200 |

## Table (5) – Comparison of results of various models without freeboard

| **Parameter** | **Proposed model** | | | | | | **Babaeyan-Koopaei et al. (2000) model** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Q = 1 $m^3$/sec. S= 0.001 n= 0.035** | | **Q = 10 $m^3$/sec. S= 0.001 n= 0.035** | | **Q = 100 $m^3$/sec. S= 0.001 n= 0.035** | | **Q = 1 $m^3$/sec. S= 0.001 n= 0.035** | | **Q = 10 $m^3$/sec. S= 0.001 n= 0.035** | | **Q = 100 $m^3$/sec. S= 0.001 n= 0.035** | |
| | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** | **Trapezoidal** | **Round-Bottomed Triangle** |
| **z** | 0.577 | 0.577 | 0.666 | 0.666 | 0.577 | 0.577 | 0.577 | 0.577 | 0.666 | 0.666 | 0.577 | 0.577 |
| **y** | 1.030 | 1.088 | 2.551 | 2.750 | 5.711 | 5.875 | 1.005 | 1.030 | 2.384 | 2.442 | 5.654 | 5.791 |
| **P** | 3.487 | 2.415 | 8.296 | 6.784 | 19.560 | 18.792 | 3.483 | 3.346 | 8.273 | 8.010 | 19.585 | 18.815 |
| **A** | 1.754 | 1.303 | 9.860 | 8.639 | 55.219 | 54.328 | 1.751 | 1.723 | 9.852 | 9.725 | 55.361 | 54.481 |
| **T** | 2.296 | 2.295 | 5.561 | 5.479 | 12.963 | 13.205 | 2.321 | 2.378 | 5.723 | 5.835 | 13.054 | 13.372 |
| **v** | 0.570 | 0.767 | 1.014 | 1.157 | 1.810 | 1.840 | 0.571 | 0.580 | 1.015 | 1.028 | 1.806 | 1.836 |
| **Cost** | 122.15 | 85.48 | 386.338 | 380.260 | 1199.200 | 1168.100 | 122.0 | 117.61 | 346.710 | 337.550 | 1141.160 | 1109.260 |