Optimization Control of DC Motor with Linear Quadratic Regulator and Genetic Algorithm Approach

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Abstract

This paper presents LQR and GA controllers which applied to control the speed of a DC motor and to maintain the rotation of the motor shaft with particular step response. In the state space, the control strategy is the states feedback and the most used techniques are the LQR. Liner quadratic regulator (LQR) provides an optimal control law for a linear system. It's a control strategy based on minimizing a quadratic performance index. In despite of the good results obtained from these method, the control design is not a straight forward task due to the trial and error method involved in the definition of weight matrices. In such cases, may be hard tuning the controller parameters in order to obtain the optimal behavior of the system. In this work, it proposes a states feedback technique in which there are no trial and error processes involved and the control design is carried out to fulfill specifications, for minimize overshoot and minimize settling and rising times. The proposed technique is based on the use a genetic algorithms. The obtained results show that is possible to design controllers which fulfill design specifications.

Keywords: DC Motor ,Optimal Speed Control , Linear Quadratic Regulator (LQR) Genetic Algorithm(GA) , MIMO System

السيطرة المثلى على محرك التيار المستمر باستخدام المنظم الخطي المتعامد والخوارزمية الجينية

الخلاصة

هذا البحث يبين تطبيق مسيطر المنظم الخطي المتعامد والخوارزمية الجينية على سرعة محرك تيار مستمر والهدف المرجو هو الحفاظ على سرعة مستقرة وثابتة للمحرك. الإستراتيجية هي مكافحة ردود فعل الاستجابة وأكثر التقنيات المستخدمة هي المنظم الخطي المتعامد. حيث توفر هذه التقنية نظام المراقبة الأمثل للأنظمة الخطية لتقليل مؤشر الأداء. على الرغم من النتائج الجيدة التي تم الحصول عليها من هذه الطريقة، نظام السيطرة ليست مهمة سهلة بسبب طريقة الخطأ والصواب التي ينطوي عليها تعريف مصفوفات الوزن. في مثل هذه الحالات، قد يكون من الصعب ضبط معاملات التحكم من أجل الحصول على السلوك الأمثل للنظام. في هذا البحث، يقترح أسلوب ردود الفعل في الحالات التي لا توجد بها عمليات التجربة في الخطأ والصواب المعنية لاختيار مصفوفات الوزن لتصميم التحكم لتحقيق المواصفات بتخطي الحد الأقصى لاستجابة والحد من تقليل زمن الاستقرار و الارتفاع

لاستجابة. وتستند هذه التقنية المقترحة على استخدام الخوارزميات الجينية. وتظهر النتائج أن من الممكن أن يحكم

التصميم ويلبي المواصفات .

الكلمات الدالة: محرك التيار المستمر ،السيطرة المثلى للسرعة، المنظم الخطي المتعامد ،الخوارزمية الجينية.

Introduction

Currently, requirements for control systems are more strict due to the need to perform more complex and higher precision tasks. In order to satisfy the increasing demands for the control design it is necessary to use modern control theory, which applies to complex multiple-input and multiple-output systems (MIMO), linear or nonlinear, [1]. The control design for MIMO systems requires the system state space representation and it uses state feedback as control strategy. The control systems performance depends on the parameters calculated through the use of optimization techniques. The most used techniques for state feedback control design are the Linear Quadratic Regulator (LQR). In despite of the good results obtained from this method, their startup realizations are not straightforward tasks due to the trial and error method involved in the definition of weight matrices. In such cases, may be hard tuning the controller parameters in order to obtain the optimal behavior of the system. In this work, it proposes a genetic algorithm (GA) whose the evolution occurs in order to attend the design specifications. Hence, there are no weight matrices to be chosen and it avoids the trial and error method in the control parameters search. The design specifications are the maximum overshoot and minimize the rise and settling time. The obtained results provided by the proposed GA are compared to the obtained results provided by the LQR technique. The proposed GA presented results as good as the LQR results, and the control parameters tuning is simpler for the proposed technique.[2]

States Feedback Control Design

Engineering systems trend is increasing its complexity mostly due to the need for performing complex and higher precision tasks. Due to the performance stringent requirements of control systems, the modern control theory and design of complex control systems have been developing since 1960. The classical control theory is applicable only for linear, time invariant, single input and output systems and works in the frequency domain. The modern control theory is applied to multiple inputs and outputs systems, linear or nonlinear, and works in the time domain. The modern control theory is based on the state concept.

State-space equations involve three types of variables present in the modeling of dynamic systems: input variables, output variables and state variables. The state equations are represented as follows, [2].

$$\dot{X} = AX + BU \quad \text{--------------------} (1)$$
$$Y = CX + DU \quad \text{--------------------} (2)$$

Where,

X is the states vector, order n;

U is the input vector; order r;

Y is the output vector, order m and

A,B,C and D are the state-space model matrices

Plant Model

The speed of a DC motor is proportional to the voltage applied to it. While, its torque is proportional to the motor current. Speed control can be achieved by variable battery tapings, variable supply voltage, resistors or electronic controls. A simple motor model is shown in Fig.1. The armature circuit consist of a resistance (R_a)connected in series with an inductance (L_a), and a voltage source (e_b) representing the back emf (back

electromotive force) induced in the armature when during rotation. [3]

The motor torque Tm is related to the armature current, ia , by a torque constant Ki;

$$T_m = k_i i_a \quad \text{---------------------------(3)}$$

The back emf, eb, is relative to angular velocity by;

$$e_b = k_b w_m = k_b \frac{d\theta}{dt} \quad \text{---------------------(4)}$$

From Fig. 1 we can write the following equations based on the Newton's law combined with the Kirchhoff's law:

$$L_a \frac{di_a}{dt} + R_a i_a = e_a - k_b \frac{d\theta}{dt} \quad \text{--------- (5)}$$

$$J_m \frac{d^2\theta}{dt^2} + B_m \frac{d\theta}{dt} = k_i i_a \quad \text{------------- (6)}$$

The DC motor data taken for this work are shown in table.1. [3]

There are several different ways to describe a system of linear differential equations. The plant model will be introduced in the form of state-space representation and given by the equations.

$$\dot{X} = AX + BU \quad \text{---------------------- (7)}$$
$$Y = CX + DU \quad \text{---------------------- (8)}$$

According to eq.s from (4) to (6), the state space model will be.

$$\begin{bmatrix} \dot{i}_a \\ \dot{w}_m \\ \dot{\theta}_m \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -k_b/L_a & 0 \\ k_i/J_m & -B_m/J_m & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ w_m \\ \theta_m \end{bmatrix} + \begin{bmatrix} 1/L_a \\ 0 \\ 0 \end{bmatrix} e_a \quad \text{---(9)}$$

$$w_m = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ w_m \\ \theta_m \end{bmatrix} \quad \text{----------(10)}$$

With the block diagram as shown in Fig. 2. [3]

Control Methods

In this paper, two control methods are proposed and explained in detail which are Linear Quadratic Regulator (LQR)and genetic algorithm (GA).Furthermore, the following design specifications have been made to evaluate the performance of both control schemes.

1) The percentage of overshot, less than 5%.
2) The Rise time (Tr),less than 1.5 second.
3) The settling time (Ts),less than 2.5 second.
4) Steady-state error is less than 2%

1. *Design Of LQR Controller.*

LQR control that designed is classified as optimal control systems. This is an important function of control engineering. Fig.3 shows the designed LQR state-feedback configuration. The purpose of the design is to realize a system with practical components that will provide the desired operating performance. The desired performance can be readily stated in terms of time domain performance indices. For example, the maximum overshoot and rise time for a step input are valuable time domain indices. In the case of steady state and transient performance, the performance indices are normally specified in the time domain.

The performance of a control system can be represented by integral performance measures. Therefore, the design of the system must be based on minimizing a performance index, such as the integral of the squared error (ISE).

The specific form of the performance index can be given as in eq.(11), where x^T indicates the transpose of the x matrix, then, in terms of the state vector, is

$$J = \int_0^{t_f} (x^T x) \, dt \quad \text{---------------------- (11)}$$

Where x equals the state vector, and t_f equals the final time.[3]

Then the design steps are as follows:

1-Determine the matrix P that satisfies eq. (12-a), where H is known.

$$H^T P + PH = -I \quad \text{--------(12-a)}$$

2-Minimize J by determining the minimum of eq.(12-b) by adjusting

one or more unspecified system parameters. [3]

$$J = \int_0^\infty x^T x \ dt = x^T(0) \ P \ x(0) \quad ----(12\text{-}b)$$

Upon examining the performance index (eq.11), recognizing that the reason the magnitude of the control signal is not accounted for in the original calculation is that **u** (equals the control vector) is not included within the expression for the performance index. However, in many cases, the expenditure of the control signal energy are concerned. For example, in an electric vehicle control system, **(u)²** represents the expenditure of battery energy and must be restricted to conserve the energy for long periods of travel. To account for the expenditure of the energy of the control signal, it will be utilize the performance index.[3]

$$J = \int_0^\infty (x^T I \ x + \lambda u^T u) \ dt \quad -----------(13)$$

Where λ is the scalar weighting factor and I = identity matrix. The weighting factor λ will be chosen so that the relative importance of the state variable performance is contrasted with the importance of the expenditure of the system energy resource that is represented by $u^T u$ as in the previous paragraphs, the state variable feedback will be represented by the matrix equation.

u= - K x --------------------------------(14)

Now, substituting eq.(14) into eq.(13), then

$$J = \int_0^\infty [x^T (I + \lambda K^T K) \ x] \ dt = \int_0^\infty x^T Q x \ dt \quad ------(15).$$

Where Q $=(I + \lambda K^T K)$ is ann×n matrix.

Postulating the existence of an exact differential so that

$$\frac{d}{dt}(x^T P \ x) = -x^T Q \ x \quad ------------------(16)$$

Then, in this case, it is required that

$$H^T P + PH = -Q \quad ---------------------(17)$$

As before in eq.(12-a)

$$J = x^T(0) \ P \ x(0) \quad --------------------(18)$$

Now, the design steps are exactly as for eq.(12-a) and eq.(12-b) with exception that the left side of eq.(17) equal –Q instated of –I. of course, if λ=0 eq.(17) reduce to eq.(12-b). and feedback

u= - K x = -[k_1 k_2 ....k_n] x.

The performance index is

$$J = \int_0^\infty (x^T Q x + R u^2) dt$$

where R is the scalar weighting factor. This index is minimized when:

K = R⁻¹ Bᵀ P

The $n \times n$ matrix p is determined from the solution of equation

Aᵀ P + PA – P B R⁻¹ Bᵀ P + Q=0 --(19)

The two matrices Q and R are selected by design engineer by trial and error. Generally speaking, selecting Q large means that, to keep J small. On the other hand selecting R large means that the control input u must be smaller to keep J small. One should select Q to be *positive semi definite* and R to be *positive definite*. This means that the scalar quantity $x^T Q \ x$ is always positive or zero at each time t. And the scalar quantity Ru^2 is always positive at each time t . eq.(19) can be easily programmed for a computer, or solved using MATLAB eq.(19) is often called the Riccati Equation. This optimal control called the Linear Quadratic Regulator (LQR) which is shown in state-space configuration in Fig. 3.Combine Fig.2 with Fig.3 yields Fig.4 which shows the use of LQR controller with the DC motor.[3]

2. Genetic Algorithm

Genetic Algorithms (GA's) are a stochastic global search method that mimics the process of natural evolution. The genetic algorithm starts with no knowledge of the correct solution and depends entirely on responses from its environment and evolution operators

(i.e. reproduction, crossover and mutation) to arrive at the best solution. By starting at several independent points and searching in parallel, the algorithm avoids local minima and converging to sub optimal solutions. In this way, Gas have been shown to be capable of locating high performance areas in complex domains without experiencing the difficulties associated with high dimensionality. A genetic algorithm is typically initialized with a random population. This population (mating pool) is usually represented by a real-valued number or a binary string called a chromosome The fitness of each chromosome is assessed and a survival of the fittest strategy is applied. There are three main stages of a genetic algorithm, these are known as *reproduction, crossover* and *mutation*. The Graphical Illustration of the Genetic Algorithm is shown in Figure 5.[4,5]

Objective Function

Writing an objective function is the most difficult part of creating a genetic algorithm. In this paper, the objective function is required to evaluate the best LQR controller for the Dc motor system. An objective function could be created to find a LQR controller that gives the smallest overshoot, fastest rise time or quickest settling time but in order to combine all of these objectives it was decided to design an objective function that will minimize the error of the controlled system. Each chromosome in the population is passed into the objective function one at a time [6].

The chromosome is then evaluated and assigned a number to represent its fitness, the bigger its number the better its fitness. The genetic algorithm uses the chromosome's fitness value to create a new population consisting of the fittest members.

The chromosome is formed by three values that correspond to the three gains of the weight matrix Q and R to be adjusted in order to achieve a satisfactory behavior is illustrated in Fig.6. The gains q_{11} , q_{22} and r_{11} are positive numbers and characterize the individual to be evaluated. The block diagram of LQR with GA controller of the DC motor is shown in Fig.7 [7].

The objective function is the calculation of its associated fitness. The fitness function is the measure of the quality of chromosome and can be defined as.

Minimize J

Where,

$$J = \frac{1}{N}\sum_{i=1}^{N} e_i$$

N=3 and

$$e_i = \{ t_s^2, t_r^2, D^2 \} \quad , \text{where,}$$

$$D = \begin{cases} (M_p\text{-}1) & \text{if } (M_{pcl}\text{-}1)^2 < 0.05 \\ \\ \infty \end{cases}$$

t_s is the settling time of GA, t_r is the rising time of GA and M_p is the maximum over shoot For GA , M_{pcl} is the maximum over shoot For closed loop response.

Simulation and Results.

The considered DC motor parameters are based on the system described in Table(1).

The best LQR controller parameters are

$$Q = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.028 \end{bmatrix} , R = \begin{bmatrix} 0.2 \end{bmatrix}$$

The elements of   K obtained by LQR method   K=[0.9749 0.8391]
and the Eigen value is

-1.8406 +1.1016i

-1.8406 -1.1016i

The time response of the closed loop system with the simulated LQR controller are shown in Fig.8

The best GA controller parameters are

$$Q = \begin{bmatrix} 53.0264 & 0 \\ 0 & 0.895 \end{bmatrix} \ , \ R = \begin{bmatrix} 52.2705 \end{bmatrix}$$

The elements of K obtained by GA method K=[0.9821 0.81]
and the Eigen value is

-1.8068 +1.1562i

 -1.8068 -1.1562i

The time response of the closed loop system with the simulated GA controller are shown in Fig.(9).

From figures 8 and 9 it can be realized that both of these controllers are suitable to utilize to control the DC motor due to both can give zero steady-state error, fast response and no overshoots at the transient response. However, the results proven that the GA method acts better than LQR controller in terms of its faster response as shown in Table (2).

Conclusions

Speed control of a DC motor is an important issue, so this paper presents a design method to determine the optimal speed control using LQR method and GA. In this paper it presented a method which uses the genetic algorithm technique applied to the state feedback control design. This method has been proposed to outline the difficult at the control design process when the LQR method is used. This difficult consists in the definition of weight matrices. Both technique have been applied to DC motor and their performance compared among them .In according to the obtained result, it noted that the proposed method presented performance equivalent to the LQR, however ,it presented the guarantee of the response to satisfy its specification for maximum overshoot and accommodation time ,since the proposed method has the design process based on these specification of the system time.

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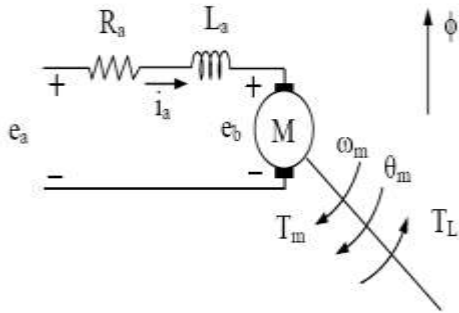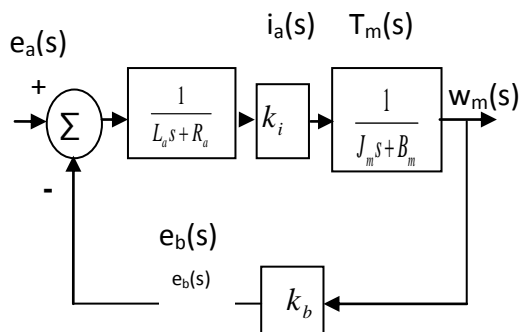
Fig.(1) DC- motor mode

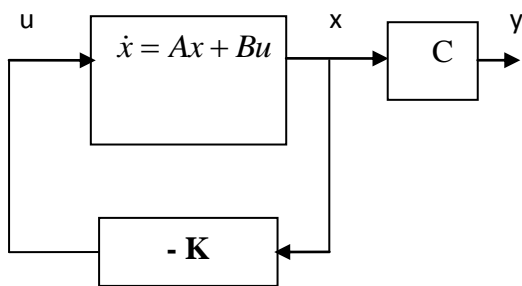**Fig. (2)** DC-Motor System Block Diagram
for speed (Plant System)
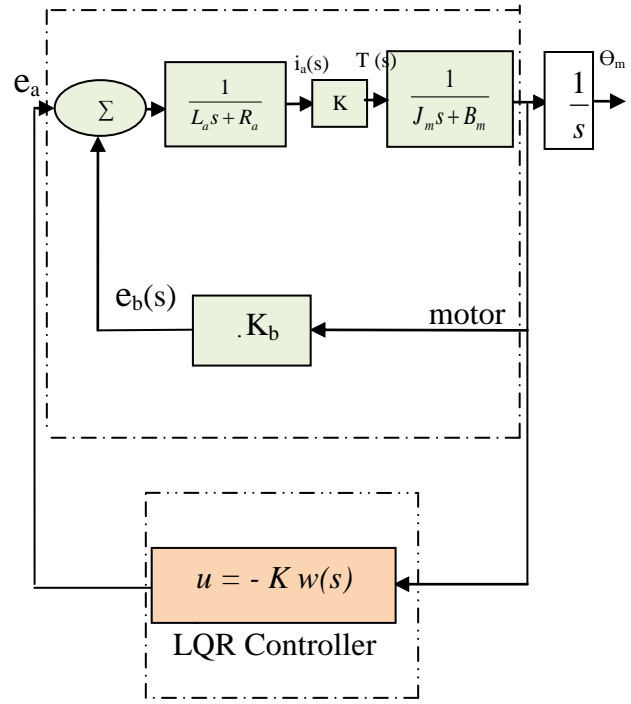
Fig.(3) Linear Quadratic Regulator
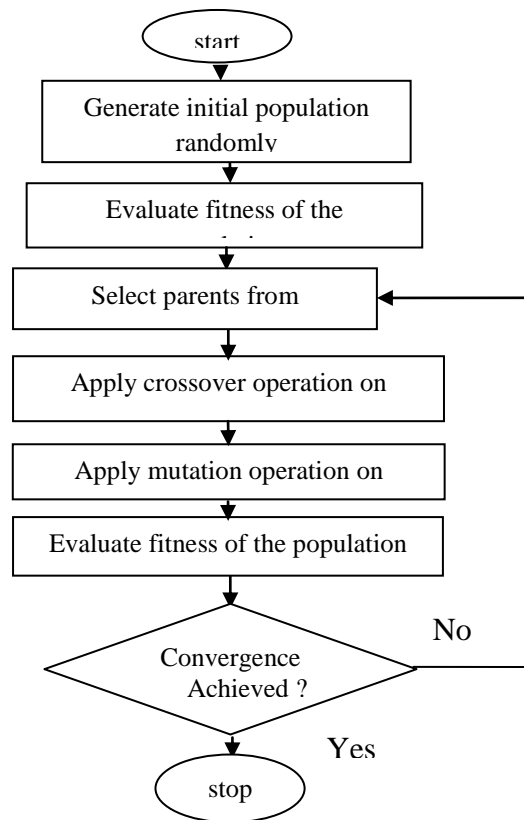Structure.

Fig. (4)- DC-Motor System with LQR

Fig. (5) Simple GA flow chart

q_{11}	q_{22}	r_{11}

Fig.(6) Chromosome Definition.
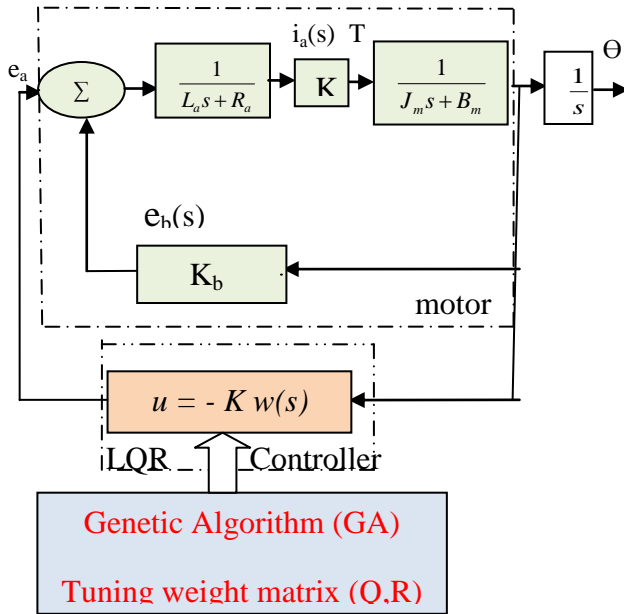
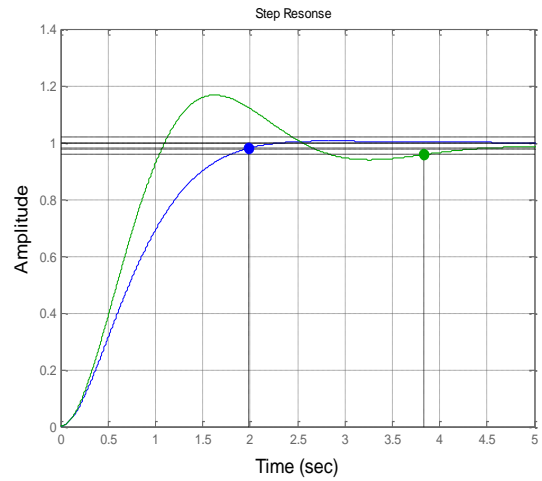**Fig(7)** Block diagram of GA with LQR controller of the DC motor

Fig.(8) Closed Loop System with the Simulated LQR Controller.

Fig.(9) Closed Loop System with the Simulated GA Controller

Table(1): DC motor parameters

Symbol	Parameter	Value and unit
e_a	armature voltage	12 volt
J_m	moment of inertia of motor	0.01kgm^2
B_m	viscous friction coefficient of motor	$0.00003 \text{ kgm}^2/\text{s}$
k_i	torque constant	0.023Nm/A
k_b	back emf constant	0.023V/rad/s
R_a	armature- winding resistance	1Ω
L_a	armature - winding inductance	0.5 H
w_m	angular velocity of the motor shaft	rad/sec
θ_m	angular displacement of the motor shaft	rad.

Table (2). Simulation Results.

Controller Type	Over Shoot	Rise Time (sec)
Without controller	19.5%	0.721
LQR Controller	0.5253 %	1.2595
GA Controller	0.7501 %	1.2225